The Importance of Diagrams, Graphics and Other Visual Representations in STEM Teaching

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Author Biography

Peter Gates has been a lecturer and researcher at the University of Nottingham in England for 25 years. He has worked in maths education for 40 years, as a teacher, teacher educator and researcher. He has a commitment to social justice and equity, and this feeds into everything he does in his professional and personal life. In recent years he has been particularly interested in working with low SES and disadvantaged young people looking to support their learning through a move away for a highly literate curriculum, looking at offering an entry to an otherwise secret garden.

Peter has two daughters, and two dogs; only two of which run to the door every time he comes home from work!

Abstract

In this chapter I look at the way we think of communication and suggest that there is an over-reliance upon linguistic and textual modes at the expense of visual and spatial modes of communication. I argue that schools fail to grasp the significance of the visual nature of communication and the implications for learning within STEM subjects. After making an argument for the importance of visuospatial forms, I provide an extensive review of the cognitive and psychological literature covering various key aspects of visualisation and how it relates to teaching STEM subjects in early secondary. It is likely that much of this will be novel to STEM teachers, yet provides us with new possibilities for opening up classroom pedagogy.

Key words

Visualisation, spatial, diagrams, graphicacy, mental representation

Introduction

> *Our research has produced convincing evidence that presenting a verbal explanation of how a system works does not insure that students will understand the explanation. In our search for ways to help students understand scientific explanations, we have come to rely increasingly on what has been called multimedia learning, through presenting explanations visually as well* as *verbally*. *Multimedia learning occurs when students receive information presented in more than one mode, such as in pictures and words. In recent years, the once near monopoly of verbally based modes of instruction has given way to the hypothesis that meaningful learning occurs when learners construct and coordinate multiple representations of the same material, including visual and verbal representations (Mayer, 1997, p. 1).*

One of the very first things a new-born learns, is to recognise faces. They have no language, know little about how the world operates, but the visual cortex kicks in with a vengeance. Within two years as the vision continues to develop toward 20/20 vision, we can not only recognise faces, but recognize ourselves in photographs, find pictures in books, recognize and match simple objects and find our way around familiar places. A child can build towers, drink from a cup, and look toward a sound. Within a very short time, an infant becomes an accomplished engineer, scientist and mathematician – all before developing language. It must be a travesty then that, a mere five years later, when the infant gets to Primary school, they find themselves bombarded with text and talk. However, when a child encounters science and technology, they are provided with frameworks for understanding and manipulating their environment. Encountering mathematics provides frameworks for making sense of the world. So the importance of learning through physical engagement, immediately encapsulates a visual element through which the leaner monitors, evaluates and hypothesizes the world.

We live in a world that is increasingly influenced by graphical and visual images, and by greater use of visual means of communication. Surely no one can fail to be impressed with Edward Tufte's work on visual display and explanations (Tufte, 1990, 1997, 2001). In a series of stunningly innovative displays he offers Byrne's presentation of Euclid's element through shape, colour and orientation (1990, p. 84-87); Snow's identification of the cause of a cholera outbreak by mapping cases (1997, p. 27-37); and Marey's description of Napoleon's devastating losses in the Russian campaign with a spatial time series graphic (2001, p. 40-41).

However, the one prominent feature of school mathematics is a dependence on *language* and *textual* communication, largely to the exclusion of other modes of communication – namely the visual. Whilst this may be partly true to a lesser extent in science, it is not the case in technology and engineering. This will derive from technology and engineering dealing directly with artefacts and "real" objects and structures, rather than the much more abstract concepts of mathematics - which do only exist in the head. There are strategies that mathematics and science can appropriate from technology and engineering, in particular broadening the modes of presentation to include the manipulation and representation of physical objects and processes. –too often missing from mathematics lessons.

However, these modes of presentation are processed quite differently in the brain with significant ramifications for classroom practice. It is recognised that *spoken language and text* are both characterised as one dimensional, sequential and sentential (Crapo, 2002) and that they are processed in the auditory centres of the brain before temporary storage in working memory (Card, Moran, & Newell, 1983). This contrasts with *visual images* which are multi-dimensional requiring processing in the visual cortex before temporary storage in working memory. Consequently, if our STEM teaching has rested upon the sequential and auditory channels, we need to rethink our approach if we are to adopt more multi-dimensional approaches to teaching and learning.

This is vital as, whilst the connection between visualization and mathematical and scientific skills is complex and contested, much evidence points to positive associations between some aspect of visualisation and spatial skills, and some elements of mathematical and scientific competencies (Cheng & Mix, 2014) and that visualisation and imagery are central to understanding and reasoning (Arcavi, 2003; Whiteley, 2004). One problem in STEM currently is …the kids just don't learn the stuff:

> *Why is it that a student can read or listen to every word of a scientific passage, including a cause-and-effect explanation, and yet not be able to use that information to solve problems? Our research has produced convincing evidence that presenting a verbal explanation of how a system works does not insure that students will understand the explanation. (Mayer, 1997, p. 1)*

The importance of visuospatial skills

How we understand the visual mode of communication is complex and involves a range of diverse elements and cognitive processes, yet evidence points to possible connections between future cognitive development and the development of early spatial skills (Kersch et al., 2008). However, this forces us to consider the distinction between how we think of "visualisation" (or its derivatives) and how that is distinct from the "spatial". There are a number of ways in which we might understand the "spatial" in STEM subjects. For example, *spatial relationships* might refer to the perception or proprioception of spatial objects whereas *spatial reasoning* would refer to a process of integration of elements into a logical connection. *Spatial visualisation* might refer to the capacity to manipulate and transform mental images, to see images within others, and to construct and dissect objects.

Despite this, our understanding of the nature and role of images in educational contexts is still limited (Postigo & Pozo, 2004). Part of this is because of the disproportionate prioritisation of much research into other modes such as language, textual or non-iconic forms. What we do know is that much of the engagement with visual material in schools is "superficial" (Postigo &

Pozo, 2004, p. 624). It might be surprising that there is still a case for arguing that this is important since as Weidenmann (1987, p. 157) said 30 years ago: "*the empirical evidence is so convincing*". He goes on:

> *Probably no other instructional device leads to more consistently beneficial results than does adding pictures to a text… There can be no doubt that pictures combined with texts can produce strong facilitative effects on learning and retention (Weidenmann, 1987, p. 158)*

As I suggested above, I would question the extent to which studies of the use of diagrams, illustrations, visuals etc. as pedagogical strategies within Mathematics and Science Education has influenced the culture of classrooms. The fragmentation of S-T-E-M education (rather than STEM) with little pedagogical or curricular cross-over results in lost opportunities for greater synergy between forms of presentation and representation.

Often diagrams and visuals in textbooks (most notably in mathematics) tend to be little more than "wallpaper" offering merely some irrelevant visual. Little thought goes into the creative epistemological design of text-visual components and how these contribute to the construction of mental models. There also appears to be a lack of attention given to the construction and manipulation of these mental models especially those drawing on visual-spatial processes. Dawe puts as an imperative for teachers to "*consciously link visual images, verbal propositions and memories of activities, involving the manipulation of physical objects*" (Dawe, 1993).

Specific visual skills useful in such activities as science and engineering might include folding, cutting and rotating (Nordina et al., 2013) – and such skills are easily incorporated as direct strategies into STEM lessons. Other visual skills useful in STEM learning might include, explicit constructing a mental image or mental model of a scientific or mathematical artefact or process, developing and using a mental representation, constructing representative diagrams, describing (representing) images and models, mental rotation – and it goes on – where the focus is on the *representation itself*, rather than the concept.

Whilst visualisation and mental imagery are cognitive processes that are evident from birth, there would appear to be different levels of individual facility; yet there is little evidence of explicit instruction at school level. One question is whether these skills are actually open to enhancement through classroom activity – but there is evidence of gains after explicit training in visualisation (Lord, 1985, 1990). The need to be proficient in visualisation is important in many fields such as engineering (Olkun, 2003; Strong & Smith, 2002), medicine, and construction - in fact it may be difficult to find a field of employment where it is not important. The lack of direct instruction in visual facility in school mathematics is therefore worrying.

Lowrie and Diezmann (2005) developed the Graphical Literacy in Mathematics Test, and subsequently studied elementary school children in Australia finding their performance was not particularly strong. They argued for much more explicit teaching of reading, producing and understanding decoding information graphics. This poses problems not only for pedagogical resources but also for current test items where the forms of graphics used may detract from the underlying mathematics bring tested producing inaccurate results (Lowrie & Diezmann, 2009).

There are further claims of gender and class differences in spatial skills (Linn & Peterson, 1985) and age effects (Bishop, 1978). Linn and Peterson argue there is evidence of males using a holistic approach, with females taking an analytical approach to visuospatial methods. Bishop argues that there is an interesting developmental process moving from topological, through 2D to increased sophistication in 3D. The widespread use of computer gaming by young people may also be enhancing their visual and spatial ability. However as with all developmental processes, environment and social factors will play a significant part. This raises a question of what pedagogical approaches and tasks best foster a growth in visual and spatial skills and what place information technology holds in that process. More particularly, we might consider how the playing of computer games can enhance understating of STEM through the visual activity inherent in engagement (Beck & Wade, 2006; Gee, 2007).

Achievement and the Visual

The prospect of teaching fractions, yet again, to a class of low achieving adolescents strikes abject frustration in mathematics teachers throughout the world and the same will be true for the other STEM subjects. Yet the reality is many young people fail to understand even basic mathematical and scientific concepts. How we get to the position after 9-10 years of formal compulsory schooling that we are still trying to convince many children that 1/4 = 2/8 is nothing short of an international scandal. Of course it is not just fractions that we fail to teach; the list goes on and on covering much of the mathematics and science curriculum. Worryingly, this is after decades of curriculum reviews, policy changes and millions spent on research.

Indeed, learning fractions provides one of the areas within mathematics curriculums around the world that persist in posing difficulties for young children (Carpernter et al., 1976; Ellerton & Clements, 1994; Neime, 1996), although in some countries (Singapore and South Korea) international comparisons – albeit notoriously unreliable - suggest children do much better (Mullis et al., 2012). One argument for the difficulty is the dependence on particular models over others (Zhang et al., 2015), notably the area model (Cramer et al., 2002) and the widespread incorporation of the dubious pizza as a model of fractions. In both these cases, the visual appears very much as an afterthought rather than a carefully designed intervention aimed at supporting robust mental representations.

By looking at *visualisation* (and visual *reasoning*) as distinct from "*spatial*" we might see the first as a feature of cognition, and the second as a feature of the physical world. Addressing the visual does not mean doing more geometry. Rather it means looking at how mental models of concepts are held and manipulated within an increasingly complex grasp of STEM subjects.

Within mathematics and science this distinction would connect most obviously into particular concept areas (such as geometry, mechanics, measurement, graphical representation) but also into problem solving as well as into perception and organization of logical reasoning. Much research has been undertaken within the area of geometrical and spatial understanding and awareness. However, measurement (in 1, 2 and 3 dimensions) also provides a context for spatial relationships to be built and developed. The area of graphical representation also necessitates a use of space to represent and manipulate relationships as well as to draw on multiple representations. Spatial visualisation would be drawn on when presented with some object and needing to see it from a different perspective (rotation, scaling, inversion etc.). In this way spatial cognition is contrasted with a more text/linguistic based mode of cognition and information processing, using analytical logico-deductive reasoning (Baddeley, 1998).

To *reason visually* - to use spatial representation to demonstrate a logical process- is maybe something not yet at the forefront of teachers' conceptions of STEM learning. Hence the use of diagrams and visuals may currently play a very insignificant role in classroom practice. Diagrams, graphics and other visuospatial representations have been an interest within mathematics and science education for some time, but are becoming of more relevance now given the increasingly visual nature of our communications mediated through technology.

It is possible that facility with using graphics (contrasted with "graphs") might contribute to successful learning (Schnotz et al., 1993). Visual information does have a number of advantages for the process of learning – notably in illustrating abstract concepts and organising complex information (Schnotz, 1993). Yet it remains questionable whether a concern for visual, mental or multiple representations has yet influenced school pedagogy and curriculum sufficiently for teachers to adopt a more multi-modal approach. This may be due to the lack of an overarching coherent theoretical framework within which such work can be situated. However, it is also likely to be influenced by the culture of STEM teaching and specifically the preponderance of propositional lexical-textual forms of argumentation. Furthermore, the culture of teaching might make it more problematic incorporating a clear framework for visual information which draws on different theoretical ideas and frameworks. Dreyfus (1991) argues there are in fact two specific issues that need understanding – (a) the difficulty of visualisation, and (b) the status of visualising within teaching identified by the low status accorded to visualisation by both pupils and teachers (Dreyfus, 1991, p. 34; Eisenberg & Dreyfus, 1991) which I discuss later.

Representation

The importance of mental representations for a learner within STEM is widely acknowledged as ways of constructing mental models of entities or processes. The process of instruction is surely to support that construction, but this needs to be done with some understanding of "cognitive architecture":

> *"Visualisation extends working memory by using the massively parallel architecture of the visual system to make an external representation function as an effective part of working memory" (Crapo et al., 2000, p. 220; citing Larkin & Simon, 1987)*

Visual representation can thus reduce the cognitive load during engagement (Clark et al., 2006). Specifically, when used in a supportive way with text (as "*spatial text adjuncts*"), visuals (diagrams etc.) can help to:

- *represent* the text, providing additional nonverbal memory prompts;
- *organise* and provide structure and form to text;
- *interpret* otherwise complex text; and
- *transform* text into pictorial images that can be stored more efficiently.

(Robinson, 2002, p. 1)

This four-fold typology can be incorporated into teaching quite easily though does require us to rethink our materials and tasks. A further typology of representation was proposed by Lesh et al. (1987) which included five elements: static pictures, manipulative models, written symbols, real-life situations, spoken language. As with any typology, this will have its limitations but offers a structure that might be useful for classroom application. Whilst it offers a typology of external representations rather than a description of internal cognitive function, it can provide teachers with a form of classification of forms of presentation they can use to analyse the balance between different modes they use in their classrooms. This would likely demonstrate that spoken language and written symbols dominate classroom discourse. However, it can also help us to ask whether static pictures match student understanding in operationalising mathematical and scientific concepts. The interplay between diagrams, students' visual models and their representation of concepts is a very complex relationship (Anderson-Pence et al., 2014). However, graphics and text are argued as *different* modes of representation and thus play *different* roles in fostering understanding. (Schnotz et al., 1993).

> *The essential point here is not only that two codes are better than one, but rather the combination of two qualitatively different principles of representation which complement one another and make possible a high efficiency of human cognition. (Schnotz, 1993, p. 248)*

Studies of graphical and textual use from a psychological perspective have already provided us with evidence that each are processed differently in the brain (Schnotz et al., 1993). Whereas text rests on symbols, drawing on propositional logics, diagrams draw on more spatial forms or arrangement, and in this way they can be seen to fit into Paivio's dual coding theory (Paivio, 1976, 1978, 1986) and Baddeley's two phase short term memory model (Baddeley, 2003; Repovš & Baddeley, 2006). In working on text, a symbolic propositional representation is constructed on the basis of the *semantic* structure, which then constructs relationships between elements that goes on to construct a mental model. With graphical and diagrammatic representations these are processed first as a *visual configuration* which then constructs an analogue mental model

> *In other words, text and graphics are complementary sources of information insofar as they contribute in different ways to the construction of a mental model. A text triggers the formation of a symbolic propositional representation which then serves as a basis for the construction of an analogue mental model. Conversely, a graphic*

> *can be considered as an external model which enables a more direct construction of a mental model via an analogue visual representation. (Schnotz et al., 1993, p. 183)*

In this way graphics can be understood as closer to the structural form of an intrinsic mental model (Schnotz, 1993) representing some concept or process. Text on the other hand does not carry such structural features.

For too long, classroom practices on use of graphics and visuals have rested upon outdated ideas drawn on an absence of knowledge of cognitive modelling. Where representations are concerned, it is almost “the more the merrier – sort of”, as Ainsworth explains:

> *Research on learning with representations has shown that when learners can interact with an appropriate representation their performance is enhanced. Recently, attention has been focused on learning with more than one representation, seemingly predicated on the notion ‘that two representations are better than one’. Yet, as research on learning with multiple external representations (MERs) has matured, it is increasingly recognised that the issue is not whether MERs are effective but rather concerns the circumstances that influence the effectiveness of MERs. … Schnotz (2002; Schnotz & Bannert, 2003) focuses not on pictures and text per se, but on depictive (iconic) and descriptive (symbolic) representations. In this approach, mapping happens at the level of mental model construction and what results is not an integrated representation but complementary representations that can communicate with one another. (Ainsworth, 2006, p. 183-4)*

Hence, as I argue elsewhere in this chapter, text and visual (or depictive and descriptive) representations are not incorporated into some complex mental model, but are held and enacted separately.

Typology of diagrams

Larkin and Simon (1987) suggest three main reasons why diagrams “can be superior to verbal descriptions”:

- Diagrams can group together information that is used together thus avoiding textual searching;
- Diagrams use location to group information avoiding a need to match symbolic labels; and
- Diagrams automatically support as large number of perceptual inferences

(Larkin & Simon, 1987, p. 98)

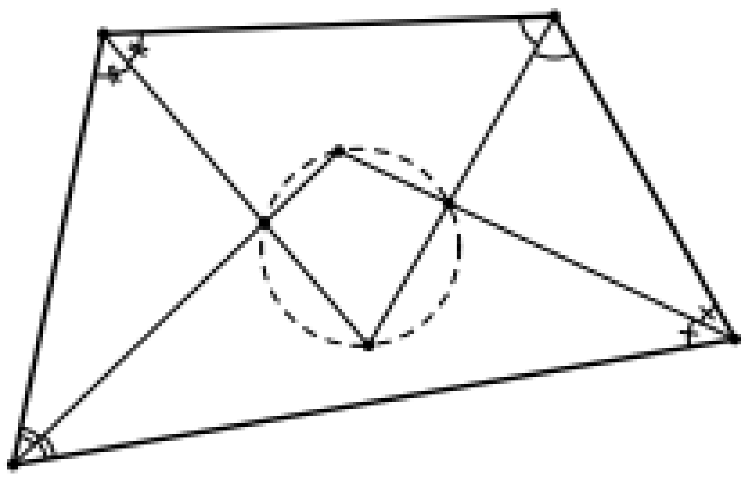
Largely then for Larkin and Simon (1987) diagrams have a distinct advantage when considering the necessary computational requirements, but users need to know the way to use this. One area of confusion is identifying a typology of visual/diagrammatic forms which is both a theoretical question (i.e. the form, structure and semantics of visual representations), but also an empirical one (i.e. what forms are recognised and used in classroom and teaching materials). Hittleman (1985, pp. 32-33) has studied the role of illustrations and explored pedagogical implications for teachers. He offered a typology of six types of illustrations in science texts, which again might be useful for STEM teachers to incorporate in teaching:

- **Photographs**
  Accurate depictions of a scene or object
- **Realistic drawings**
  Generally related to the object
- **Representational drawings**
  Less accurate that realistic such that some elements or features are highlighted
- **Diagrams**
  Representational but drawing on symbolic representations identifying relationships may focus on particular elements of a whole

- **Charts, graphs and figures**
  Information organised spatially, representing relationships in various ways
- **Maps**
  Representation of some physical reality with some topology or spatial structure.

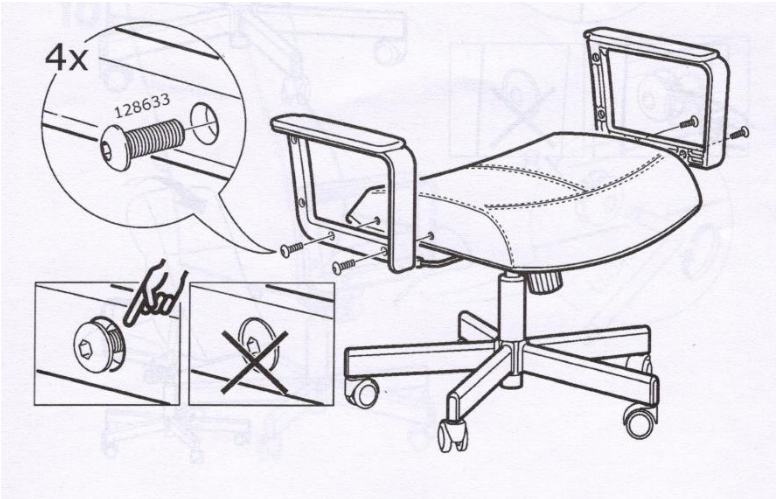
It seems reasonable for such a typology to apply to all STEM texts and resources, though no formal analysis has taken place. Importantly "*when they learn to read illustrations, [children] need to understand the various signals illustrators use to convey meaning*" (Hittleman, 1985, p. 33). The six-fold typology above provides several different *signal systems*, and inducting children into these rules and conventions is likely to put them in a better position to use and interpret various modes of communication.

How do visuals convey meaning and how can we help children interpret visuals? Different types of information are stored and processed differently in the brain – some as image-like structures (Kosslyn, 1980), as a result certain styles of material are rendered easier to learn by representing in graphic form presenting concepts available for simultaneous processing. Visuals can convey meaning, but not arbitrarily so; rather through conventions and sometimes specific systems of logic (Winn, 1987) and the exploitation of use of space – arrangement, structures, linkages, graphic forms convey meaning differently and learners do need to be introduced to the conventions. A nice example of the use of visual systems is in *Geometry in Figures* (Akopyan, 2011). This book consists of some 130 pages packed with diagrams with hardly any words in the whole book. It is the job of the reader to derive the geometrical proof and statements without any text. The following is a nice example (Akopyan, 2011, p. 7). The grammar has to be derived, but is relatively simple.



You might at this stage try to put into words the property that this diagram demonstrates – and how this is extracted from the visual grammar (I provide the solution at the end of the chapter).

Here is another example from an IKEA manual on how to put together a chair. The grammar here is very simple, depending on a ☜ and a **X**.



Text and Graphics

Diagrams do not "*speak for themselves, but are read*" (Roth, 2002, p. 20). Consequently, as with all texts, they are interpreted within a semiotic framework of meanings. Brna et al. (2001) have argued that using and reasoning with diagrams depends on the specific task, the semantic properties of the diagram and the prior knowledge of the learner (p. 116). So a graphic might not just be an advantage but might also pose difficulties because a learner needs not only to have some grasp of the underlying concept, but also be aware of the semantic visual components of diagrams which need decoding. These may be the deeper source of problems rather than misconceptions or lack of understanding of mathematical and scientific concepts (Roth, 2002).

Schnotz's study suggested that contrary to previous claims that graphics were only a secondary subordinate representation, successful learners used graphics and text in mutually constructive (adjunct) ways, and that when supported and elaborated by text, graphical and visual forms contribute to more robust and effective learning. Stone and Glock (1981) also examined a very specific context of university undergraduates using simple perspective line drawings and directions for assembling a model. However, what their work suggests is that there is certain information that is best processed visually. The work of Stone and Glock (1981) illustrates a situation where studies look into very specific psychological contexts, but which provide only very limited support for the design of secondary classroom approaches. They do suggest complementarities come partly through the use of text and visual *together* to resolve ambiguities in each modality.

Mayer and Gallini (1990), in a study of illustrations in learning science, conclude that illustrations or diagrams are effective when both the text and illustrations are "appropriate" for the task. In their case, were text was explanative (rather than descriptive or narrative) and diagrams represented both the structure and the dynamic of the instruction. In this way we can see the structure/object and process/dynamic elements that run through mathematics and science. Their work raises the possibility that illustrations should explicitly help the learner construct workable mental models (Mayer & Gallini, 1990). Interestingly their work raises another possibility - that explanative illustrations which embody the processes behind a mental model are more effective in engendering understanding in problem solving, but not in verbal recall – specifically with "low prior knowledge students". Overarching this work is that the visuals are rather more than simple pictures.

In reviewing the literature on illustrations, Carney and Levin (2002) argued that "*carefully constructed text illustrations generally enhance learners' performance*" (p. 5) and offered ten commandments (from Levin *et al.*, 1987) for teachers using illustrations (pp. 20-22):

1. Pictures shalt be judiciously applied to text, to remember it wholly.
2. Pictures shalt honor the text.

3. Pictures shalt not bear false fitness to the text.
4. Pictures shalt not be used in the presence of "heavenly" bodies of prose.
5. Pictures shalt not be used with text cravin' for images.
6. Pictures shalt not be prepared in vain.
7. Pictures shalt be faithfully created from generation to generation.
8. Pictures shalt not be adulterated.
9. Pictures shalt be appreciated for the art they art.
10. Pictures shalt be made to perform their appropriate functions.

Eitel and Scheiter (2015) present a review of 42 studies into the sequencing of text and illustration, concluding that the complexity of the content alone should determine the sequence of presentation. Scaife and Rogers (1996) discuss diagrammatic representations arguing that research "*supports for the important role of diagrams as external memories, enabling a picture of the whole problem to be maintained simultaneously whilst allowing the solver to work through the interconnected parts*" (pp. 193-194). Though it is apparent that cognitive science still has not provided a clear account of the neurological processes underway.

The importance of diagrams has been illustrated by various studies in cognitive psychology (for example through the work of Hegarty, 1992) and in mathematics education (see: Clements, 1983; Lean et al. 1981; Presmeg 1986). These suggest a significant connection between diagram use and facility with problem solving, but only where the diagrammatic form is *representational* or *schematic* - where the visual in some way represents or models the mathematical or scientific concepts, rather than merely *pictorial* (Garderen et al., 2014; Hegarty & Kozhevnikov, 1999; Stylianou, 2013).

Diagrams may be presented to learners in textual material and in tasks, yet in our very visual world, images exist all around us, resulting in young people being constantly subjected to a huge array of still and moving images, icons, photographs and representations. A very common form of these within mathematics education includes pictorial images intended to provide interest or make the material more attractive. However, the studies by van Garderen et al. (2014), and Hegarty and Kozhevnikov (1999) suggest this might have some significant yet unintended disadvantages particularly when learners are encouraged to think of diagrams as pictures rather than engaging with visual representations as an epistemological and pedagogical device. This would seem to be the space where most difficulties emerge around facility with visual representations, partly because the expectation is for a diagram to be non-representational.

The benefits of using diagrams is supported by a number of elements deriving from the cognitive engagement with multiple representations and the specific affordances offered by a visual form - specifically, identifying the structure of a problem and the interconnections between elements. Diagrams also provide a means of communication between learners and between learner and teacher. Whilst this might suggest a somewhat static representation, another benefit is through illustrating the mechanism behind a problem or the problem solving process (see Stylianou, 2002, 2010, 2013; Stylianou & Silver, 2004). Such a use as this might be termed *justificatory* (Stylianou, 2013).

However, Scaife and Rogers (1996) working within a cognitive science framework identified "a fragmented and poorly understood account of how graphical representations work, exposing a number of assumptions and fallacies" they argue for "research into graphical representations that is based on an analysis of interactivity and, thus, considers the relationship between different external and internal representations" (p. 210).

One hypothesis is the "*perceptual chunking hypothesis*" whereby skilled technicians are able to grasp whole chunks of circuit or representational diagrams as one entity, in the same way skilful chess players can "see" the whole board (Egan and Schwartz, 1979, p. 149).

Diagrams can thus be used as a record, as a means of communication, as a tool for doing mathematics and science or working on problems, and as a device for conceptual development through mental representational forms. However, a root purpose for presenting a visual form is to offer representation in different forms such learners can discern "*the common elements in* *many different embodiments of the same mathematics*" (Dienes, 1960, p. 8) so through this the learner can "*become aware of the essential sameness of the structure*" (p. 42).

Graphical understanding

Apart from geometrical understanding, one other clear area of the STEM curriculum that weaves visual processing into conceptual development is the use of graphical representations. Often the process of decoding of visuals or graphics is overlooked, yet there is evidence of the complexity posed by presenting visual graphics to learners. Graphical representations are not only functional representations within mathematics, but also exist in the world outside the classroom. They are used for a variety of reasons (See the work of Tufte for example) and subject to a variety of rule sets and conventions. Graphs have particular visual properties, but learners do not come without previous readings. Visual representations are now so widespread and profound that we are no longer aware of them (Roth, 2002). Roth offers a semiotic approach to understanding graphical literacy rather than from the stance of them as mere representational or cognitive forms based on decoding separate elements. He argued for problematizing the need to structure the visual field and to repeatedly shift back and forth between sign and referent (p. 4). However, in use, competent graph decoders "look at graphs and, without hesitation, see in each wiggle corresponding state in the world". One comes into "symbolic contact" with the phenomenon (Ochs et al., 1996)

> *Thus, when readers are very familiar with a sign system and the things it refers to, signs themselves become transparent. Readers no longer think of words, or parts of a line curve, but go directly to the things they know them to be about. This transparency is so pronounced that readers forget the distinction between sign and referent; they confuse the map with the territory (Bateson 1980, Foucault 1983). A graph simply provides the material ground that organizes competent reading; but the graph also requires competent reading to be understood and a familiarity with the situations or type of situations to which the graph refers. It is in that disappearance of the sign, the leap beyond the material basis of the text, that reading achieves its social character (Livingston 1995). (Roth, 2002, p. 6)*

However, when learners are inexperienced we can expect to see the equivalent of spelling out words, looking literally rather than looking for the meaning of the whole. Misreading distance-time graph as a trajectory for example.

> *Texts, therefore, do not speak for themselves, for they depend on a reader's familiarity with the content domain and cultural conventions regulating the signs that make up the text. (Roth, 2002, p. 15)*

In working on graphs, there are several stages one goes through (Carpenter & Shah, 1998, p. 76; Shah & Hoeffner, 2002, p. 66):

(1) the characteristics of the visual display;
(2) the viewer's knowledge of graphical schemas and conventions; and
(3) the content of the graph and the viewer's prior knowledge and expectations about that content

This final stage refers not only to decoding elements of the graph legends and label, but being able to imagine and live in the context (Carpenter & Shah, 1998):

> *Processing a geographical map or a graph involves decoding this information by learning the codes underlying it. But in addition to this syntactic component, a further requirement is knowledge about the represented content (for example, geographical, in the case of maps) that is involved in drawing inferences, which means higher-level interpretation. In other words, interpreting a map or a graph involves describing (saying what we see, observing its distribution, or following its profile), but also explaining the reason for the configuration or profile, and the degree of elaboration will depend on the subject's knowledge (Postigo & Pozo, 2004, p. 627)*

Much material in graphicacy ignores the physiological processes in encoding graphs which are described by Shah and Hoeffner (2002) – what the eyes look for and how the brain processes what the eyes see. Specifically, they compared focussed pattern spotting with an integrative model of interpretation, finding the integrative model best supporting their data.

> *Elementary-aged students are perhaps the most influenced by a graph's content. One common error is that viewers interpret abstract representations of data as an iconic representation of a real event (Bell and Janvier, 1981; Janvier, 1981; Leinhardt et al., 1990; Preece, 1990). For example, students might misinterpret a graph representing the speed of a racecar to mean the position of the racecar on a track (Janvier, 1981). This error is particularly common in contexts for which there is an obvious iconic interpretation, usually when the graph is meant to represent change (such as growth, speed) and the concrete interpretation is the value on some dimension (such as height instead of growth, location instead of speed). Although young graph readers (until around fifth grade) make this error frequently, minimal graphing instruction helps viewers overcome this error (Leinhardt et al., 1990). (Shah & Hoeffner, 2002, p. 61)*

Shah and Hoeffner (2002) offer several implications for teaching graphical literacy: *translating between representations, explicitly focusing on the links between visual features and meaning*, *making graph reading metacognitive.*

Limitations of diagrams

However, do not let us run away with the idea that visuals and diagrams are the new Jerusalem. They do have their limitations, as a pointed out by Satoy (2004) and Tversky (2010):

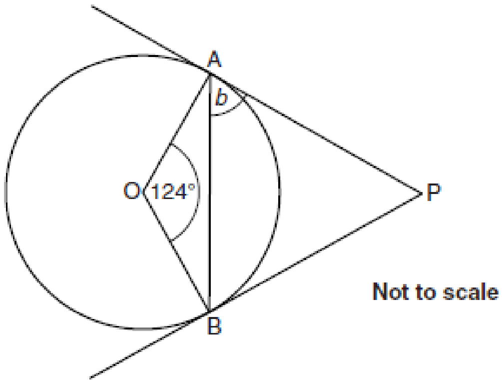
> *[Gauss] was aware that pictures in mathematics were regarded with some suspicion during this period. The dominance of the French mathematical tradition during Gauss's youth meant that the preferred pathway to mathematical world was the language of formulas and equations. […] For several hundred years, mathematicians had believed that pictures had the power to mislead. After all the language of mathematics had been introduced to tame the physical world. (Satoy, 2004, pp. 69-70)*

> *Mismatches between the natural interpretations of lines as paths or connections and the intended interpretations in diagrams turn out to underlie difficulties understanding and producing certain information systems designs. … the visual trumps the conceptual and misleads. (Tversky, 2010, pp. 21-22)*

> *Another important role for visualizations of thought is to clarify and develop thought. This kind of visualization is called a sketch because it is usually more tentative and vague than a diagram. Sketches in early phases of design even of physical objects, like products and buildings, are frequently just glyphs, lines and blobs, with no specific shapes, sizes, or distances (e.g., Goel, 1995; Schon, 1983). (Tversky, 2010, p. 25)*

There is also work from a philosophical, epistemic standpoint examining the visual within mathematics and science. One standpoint is to argue that visual representations and visualizing is not the space within which mathematics takes place. An alternative viewpoint has been argued by Giaquinto for some time (Giaquinto, 1992, 1993, 1994, 2007) that visualisation has a part to play in the construction, argumentation and comprehension of mathematics, but also that diagrams too have a significant role. In some cases, the visual and the diagrammatic present some overlapping superfluity, as the following example from a GCSE paper shows:

(b) In the diagram, A and B are points on the circumference of a circle, centre O.
PA and PB are tangents to the circle.

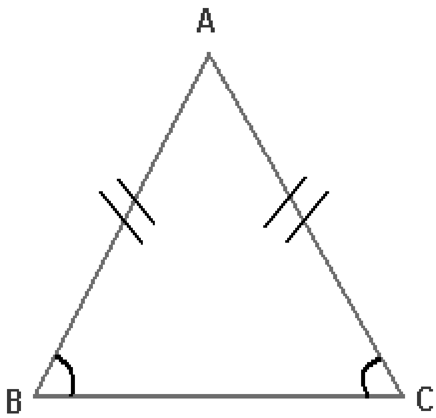


Calculate angle *b*.

Do we need the text *and* the diagram, which both give the same information? Arguably yes because the diagram makes the location of both angles 124° and b° easier to describe. The role played by the common "*Not to scale*" is important in a pedagogical as well as epistemological point of view. A diagram has the appearance of fixing certain properties. Take the following for example:

"Let ABC be an isosceles triangle"

You will no doubt have an image already, and it may be something like this:



Indeed, if you Google "*isosceles triangle*", over 90% of the images retrieved have the base angles equal. However, you might have thought of something similar when asked:

"Let ABC be an equilateral triangle"

How do we know that an image (akin to a "fixed photograph") actually represents what and only what we want to represent, in a generic way? Some argue that philosophically, you can't.

> *The danger of diagrams is that they may too easily tempt one to make unwarranted generalizations, as one's thinking may too easily depend in an unnoticed way on a feature represented in the diagram that is not common to all members of the class one is thinking about. (Giaquinto, 2007, p. 77)*

This is not a trivial pedagogic problem; there will be many learners who have developed misconceptions due to seeing the particular in the general *as* the general. Whilst there are such limitations in diagrammatic representation, in presenting the general in the particular, we have also to be open to the possibility that language in the form of text (words and symbols) has its own limitations.

Teaching to be visual in the classroom

Robinson points out "*the information that is presented to students in classrooms appears in the form of words rather than pictures*" (Robinson, 2002, p. 1). Pictures are used by teachers and in textbooks largely as illustration or decoration which is not surprising given the absence of a widely accepted framework for the use of visuals in learning.

One very particular form of visual material discussed in cognitive psychology is a *spatial-text adjunct (STA)*, a visual specifically used to support and associate with some text. Robinson suggests the potential of STAs seems to be overlooked (Robinson, 2002), but they can be used as "static and animated illustrations, geographic and knowledge maps, and graphs" (p. 3), in other words they have potential as diagrammatic, videographic, spatial, topographic and graphic.

One feature of the failure of learners to achieve, is the observation that school achievement is not equitably spread throughout society; children from less affluent homes do disproportionately worse than those bought up in relative affluence. Such children are at risk of sustaining a weak conceptual grasp of mathematical and scientific concepts and in numerical procedures, which hold them back from developing a more sophisticated understanding of STEM. This in turn closes off pathways to many careers and professions, but worse, develops into anxiety and rejection of mathematics in particular, contributing instead to an identity of "I just can't do maths" (Gates, 2001).

Whilst much research has attempted to articulate this relationship, much research has simply ignored it, either through denial, or in the belief that by providing good research all will benefit. Indeed, an early finding from the ICAAMS study (*http://iccams-maths.org/)* is that over 30 years:

> "attainment has not changed very much [..]. The general trend is for results to be somewhat lower than in the 1970s, although there are some exceptions to this. (Hodgen et al., 2010, p. 8)

So, instead of trying to do the same old thing better, maybe it is time to think anew. There is some doubt that the improvements in levels of achievement in mathematics and science in the UK trumpeted by successive Government have in reality been that real (Dickinson et al., 2010) and undoubtedly the same holds true in other countries.

The issue of prior achievement features in the visual literature to suggest there are specific benefits in using visual forms of representation when students have experienced difficulty in their prior learning or have weak verbal skills – two features which correlate highly with a learner's SES. Indeed, Arcavi suggests it might be problem with a lack of visualisation skills which can offer an explanation for many students' particular difficulties with fractions (Arcavi, 2003). This is further supported by a recent study by Moyer-Packenham et al. (2012) who reported an investigation the use of static and dynamic images with low achieving students. Although this was an action research study by just one team, low achieving students did appear to make gains in fraction leaning when provided with either virtual (computer) manipulatives or pictorial models.

A study by Mayer (1997) suggested that learners with low prior knowledge (or "*low domain knowledge*") might be particularly supported by visual models.

> *Students who possess high levels of prior knowledge will be more likely than low prior knowledge learners to create their own mental images as the verbal explanation is presented and thus to build connections between verbal and visual representations.*

> *In contrast, students who lack prior knowledge will be less likely than high prior knowledge learners to independently create useful mental images solely from the verbal materials. Thus, low prior knowledge learners are more likely than high prior knowledge learners to benefit from the contiguous presentation of verbal and visual explanations. (Mayer, 1997, p. 15)*

This is further supported by Schnotz and Bannert (2003) who studied the effect of mixing text and graphics on university students' understanding of a text. They argue that whilst "*adding pictures to a text is not always beneficial … pictures facilitate learning only if individuals have low prior knowledge and if the subject matter is visualized in a task-appropriate way*" (pp. 153-154).

> *From the perspective of practice, the findings of our study emphasize that in the design of instructional material including texts and pictures the form of visualization used in the pictures should be considered very carefully. The question is not only which information is to be conveyed. One must also ask whether the form of visualization used in the picture supports the construction of a task-appropriate mental model. Good graphic design is not only important for individuals with low prior knowledge who need pictorial support in constructing mental models. Well-designed pictures are also important for individuals with high prior knowledge because these individuals can be hindered in their mental model construction through inappropriate forms of visualization (Schnotz & Bannert, 2003, p. 154).*

Mayer takes this further looking at those learners identified as "poor readers", who may be so because of an imbalance in text vs. visual processing, and would this benefit from a visual approach:

> *Previous research on children's processing of narrative texts has shown that the poor readers profit generally more from text illustrations with regard to comprehension and learning than good readers (Cooney & Swanson, 1987; Levie & Lentz, 1982; Mastropieri & Scruggs, 1989; Rusted & Coltheart, 1979). This suggests that poor readers are able to construct a mental model from a text with pictures, whereas they would fail on the basis of a text alone. Similar results have been found for adult learners' processing of expository texts. Learners with low prior knowledge benefit from pictures in a text, whereas learners with higher prior knowledge seem to be able to construct a mental model of the described content also only from the text (Mayer, 1997).*

Furthermore, O'Donnell et al. (2002, pp. 78-79) argue that knowledge maps (concept maps) are particularly useful for learners with low or weak verbal skills, though this was probably involved more than just the use of a diagrammatic representation. Their suggestions include greater integration of map and text, collaborative construction of maps, explicit work on the isomorphism between map and text as well as providing exemplar maps.

A study on low attaining learners concludes "*reasoning with a diagram is a difficult process that students may need more time and experience to develop*"(Garderen et al., 2014, p. 147).

Hittleman (1985) argues for instruction to include a process of translation between various illustrative representation and text. There are, he argues, specific reasons why certain learners might specifically require attention in grasping the coding of illustrations:

> *Children often may experience problems in reading content area illustrations because of their need to keep switching from reading the text to examining the illustrations. This punctuated or staccato reading pattern breaks up their continuous flow and processing of information. Children who experience reading difficulty in general, and* those who are handicapped by *breakdowns because of limited ability in conceptualizing may be confused for two reasons: 1) they lack an understanding of the nature of the information in the illustrations; 2) they are hindered by an inability to translate information from one form and organizational pattern to another.*

> *Therefore, teachers should examine and orally discuss illustrations with children before they are asked to read the accompanying text. (Hittleman, 1985, p. 34)*

Hittleman problematizes the "picture is worth a thousand words" dictum by pointing out that one needs to know the words, and how to translate between the different languages:

> *Reading is an interaction between children and authors. Too often, visual displays are considered easier to read than prose because they do not entail any words. As demonstrated, however, children's reading of illustrations requires skills that must be taught and learned. Illustrations are only representations of life, and children must learn how the illustrations picture actual objects and events. Children, especially children with difficulties manifested by disabilities in cognition and learning, need direct instruction so they can understand how a three-dimensional world is represented in a two- dimensional presentation. They need to learn how to translate those representations into spoken and written messages. A picture is worth a thousand words only if the observer (reader) already knows what those words are and has skills to relate them to the picture. (Hittleman, 1985, p. 36)*

Much evidence suggests less attention is paid to visual forms in teaching, (Larkin & Simon, 1987) and this results in learners misusing, or not drawing on, visual models. More specifically within the mathematics education literature, there is some evidence that how students visualise mathematical concepts, *"plays a pivotal role in how well students apply their …understanding in novel situations*" (Anderson-Pence et al., 2014, p.???). Diagrams, in the form of static images which embody fixed images of concepts, can be crucial in supporting or hindering students' interpretations of a problem or piece of mathematics. However, diagrams are not merely accurate and unambiguous representations of mathematical objects or concepts. They require interpretation to manipulate, generate or employ in doing mathematics. For not only do diagrams embody features of mathematical concepts, they also embody aspects of pupil misconceptions (Anderson-Pence et al., 2014, p. 14).

> *The use of visualization requires a specific training, specific to visualize each register. Geometrical figures or Cartesian graphs are not directly available as iconic representations can be. And their learning cannot be reduced to training to construct them. This is due to the simple reason that construction makes attention to focus successively on some units and properties, whereas visualization consists in grasping directly the whole configuration of relations and in discriminating what is relevant in it. Most frequently, students go no further than to a local apprehension and do not see the relevant global organization but an iconic representation. (Duval, 1999, p. 14)*

> *Learning mathematics implies the construction of this cognitive architecture that includes several registers of representation and their coordination. Thus geometrical figures used to solve problems involves some ability in operative apprehension and awareness of how deductive reasoning works. Students do not come into such apprehension and awareness by themselves. Moreover, some coordination is required between operative apprehension, discursive apprehension and deductive reasoning. In other words, geometrical activity requires continual shifts between visualization and discourse. In order to achieve such coordination another kind of visualization is required. (Duval, 1999, p. 22)*

Visual representation can draw on cognitive skills that are underused elsewhere in schools. Forty years ago, in the US, Olsen (1977) argued that schools are biased toward verbal and textual forms. Consequently, school pedagogies may privilege certain learners - those confident and at ease with literal forms (Winn, 1987). In many but not all cases, "graphics have done more to improve the performance of low-ability students than those of high ability" (Winn, 1987, p. 169), particularly in science (Holliday et al., 1977) and mathematics – where it is claimed that visuals reduced "the reading-related working memory overload in poor readers" (Moyer et al., 1984).

Though there is a claim that low-ability learners have particular difficulty with materials that is informationally rich and with redundancy (Allen, 1975).

Unfortunately, teachers don't seem to have developed the same level of appreciation. In a study of assessment practices of 11 secondary mathematics teachers, Morgan (2004) argues that whilst the teachers acknowledged the importance and value of visual representations in mathematics, they often gave them a low value within pupils' work in contrast to more abstract but non-visual (re)presentations. Indeed, she went further to argue that at times teachers would assess a piece of work more negatively if a diagram was inserted as an indication of "a concrete, practical approach rather than a more prestigious abstract approach" (Morgan, 1999).

The reluctance of learners and teachers to use diagrams, as reported by Morgan and others, is likely to be influenced by the considerable difficulty posed by working with visual representations. The mathematics itself first needs to be *decoded* (Garderen et al., 2014, p. 136) from the original textual form and then *recoded* into an appropriate visual form. This is by no means trivial and requires a sense of what information is of relevance amongst information provided (*identification* and *selection*), what *connections* exist between elements of the problem, and how mathematics can represent the *structure* of a problem.

Garderen et al. have described an underlying set of skills in the context of diagram proficiency (Garderen et al., 2014, p. 137):

- To know what a diagram is and can illustrate;
- To use a visual representation to depict key component of a problem;
- To know how to represent the processes and relationships within a problem in visual form and flexibly adapt to different problem formulations; and
- To be disposed to use a visual form

This final skill is more than just an encouragement to "draw a diagram" but is a way of encouraging the learner to "see things" is a different way. It surely can be no surprise that learners who are presented in lessons with textual information day in day out, become reluctant to use diagrams, especially if (as Morgan reports) they will get criticised for doing so. Diezman et al. (Diezmann, 1999, 2000; Diezmann & English, 2001) have written of problems in diagram use such as:

- Not using a diagram at all;
- Using a diagram that was virtually unviable and unusable;
- Using a diagram that is imprecise, missing constraints of the problem; and
- Producing inaccurate diagrams due to not noticing salient features, and maybe not even being aware of salience

A study of diagram use to solve word problems (Garderen et al., 2014) also reported differential forms of engagement between higher ability learners and those with "learning disabilities". Garderen et al. (2014) went further to argue that in their study, students did not see the point of diagrams and claimed they did not even know what the term "diagram" meant, as well as not being aware of the diversity of visual representational forms. Furthermore, they did not see diagrams as part of doing mathematics. This is likely to derive from the forms used in school and elsewhere, limiting learners to restrictive visual representations e.g. being told a fraction is a pizza slice, or some shaded-in shape within a triangle. This all resulted in learners not choosing to use a diagrammatical form. Garderen et al.'s argument is that weaker pupils engaged differently with diagrammatical forms through not knowing what a diagram was, was for, how it was created, or was used. This might be an example of a broader problem in the process of representation in learning. If a student cannot see the structural relationships between the representation and concept being targeted, then any representation will remain rather meaningless.

However, further evidence indicates that there is a lack of explicit instruction in dealing with graphics - that unsuccessful learners would benefit from support and guidance in mapping between graphic and text information and the resulting mental models (Schnotz et al., 1993). A US study of 13 RCTs on learning difficulties in mathematics (see Gersten et al., 2009, p. 30 for a full bibliography) reported empirical support for using visual representations with learners who were achieving poorly in mathematics even if this was cited in some studies as providing only "*moderate evidence*" (Gersten et al., 2009, p. 30). They placed visuals explicitly within a framework consistent with Bruner's enactive, iconic, symbolic representation situated specifically between physical manipulatives and abstract symbolic representations. In this way, diagrams and visual representations should be used specifically to support learners' reasoning through transitions between physical models and symbolic representations. It is further argued that student understanding of these transitions can be strengthened through the use of visual representations of mathematical concepts (Hecht et al., 2007).

> *A major problem for students who struggle with mathematics is weak understanding of the relationships between the abstract symbols of mathematics and the various visual representations. (Gersten et al., 2009, p. 30)*

They go on to argue that materials specifically for pupils with difficulties, "*provide very few examples of the use of visual representations"* (p. 36). We can see the same reluctance to place visual reasoning in reteach studies examining instructional strategies – for example Darch, Carnine and Gersten (1984) who offer "*explicit instruction*" with no attempt to consider any visual forms between word problems and solution.

A conclusion for mathematics educators is to foster an approach with teachers to recognise and respect the visual and diagrammatical form as a pedagogical tool to represent and work on mathematics. Low attainders seem to have greater difficulty seeing the salience in a problem than can be represented in multiple ways - particularly the visual - or even to have a disposition so to do. They conclude "*reasoning with a diagram is a difficult process that students may need more time and experience to develop*"(Garderen et al., 2014, p. 147). In addition, we do not have an understanding of the way in which diagrammatic competence develops over time, maybe because we have little idea of what we mean by diagrammatic competence and have rarely used it as a legitimate pedagogical device within mathematics. For it is only once we recognise the difficultly and "*lack of transparency ... Can we begin to identify and adopt strategies to support students*" (Rubenstein & Thompson, 2013, p. 550).

Schnotz argues that a mental representation is never a representation *per se* but is dependent on the context within which the model was created and the purposes for which it is used. As a result, the use of external representations does need to "*take into account the interplay between the representation and the task demands*" (Schnotz 2002, p. 104). Descriptive (made up of symbols with an arbitrary and conventional connection to the content) and depictive representations (iconic signs associated with the content through structural features) thus have different functions and purposes within mathematics. For example, "the angles in a triangle add up to 180°, compared to a diagram in which the three angles just happen to add to 180°". However depictive representations can carry much more information and reduce cognitive load. Given this, the use of diagrams and visual representations within mathematics teaching rests upon complex cognitive operations. Yet how they feature in pedagogy is largely unknown, with little work in to how visual modalities are used by teachers, to what end. Furthermore, there is little theoretically sound examples of where visual modalities can be used to improve the quality of mathematics teaching.

Schnotz describes the ways in which learners engage with descriptive and depictive representations:

- **Textual representation** - First one is presented with linguistic information, in the form of words and symbols, from which specific syntax needs to be understood. Then semantic content needs to be interpreted and mapped onto the referential content. To make sense of this the leaner relates to their own domain specific world knowledge. Thereafter there

comes a stage of communicating the content and an appreciation of the forms of communication
- **Picture representation** – First one encounters a visual stimulus that one encodes the perceptual surface structure, drawing on common structural features between the picture and the reference. Next, one encodes the information carried within the stimulus by drawing on one's awareness of pictorial communication

However, the interplay and specific ways in which the learners enact these operations is still a matter of conjecture within cognitive neuroscience (Schnotz 2002). Anderson-Pence et al. (2014) argue that merely presenting learners with "visual static models" does not guarantee that they will be able to use and incorporate the model into their own understanding. Notably these diagrams will not necessarily expose misconceptions, or allow learners to focus on their understanding sufficiently enough to use the diagram to work on mathematics. This needs to be incorporated into instructional resources that support learners to use, develop and adapt diagrams, which in turn improves their visualisation skills (p. 14).

What people attend to makes a big difference to how they interpret and what they notice (Whiteley, 2004). Whiteley (2004) discusses the processes used in proving, within geometry, and highlights several strategies that experts use which may not be accessible to the novice.

> *Of course the expert has learned to do the animations and sequences in the mind's eye, with shifting attention, with mental movements and comparisons of pieces, with the shifts from parts to whole and back again... Too often we do not teach the skills or even explicitly model the skills in a way that the apprentices can observe and imitate. (Whiteley, 2004, p. 290)*

We can, as teachers, influence what learners attend to. This being the very purpose of teaching.

Overview

In this chapter I have underlined the importance of engaging with visual models as part of a process of learning where text and visuals are both key elements of the way our brain interprets and manipulates information. One root cause of much underachievement in STEM in early secondary school can be traced back to a heavy reliance upon textual and lexicographical representation and communication at the expense of more visuospatial representations. By recognising the diversity of use we can make of visuals and diagrams we can begin to look for a range of ways of incorporating both in classroom materials and tasks such that we place more emphasis upon the cognitive re-constructing of STEM ideas using cooperation between different forms. Such cooperation might be facilitated by greater opportunities for cross-over CPD between teachers of each component of STEM, such than divergent forms of representation along with differing incorporation of visual forms might help all STEM teachers broaden their awareness of the importance of connecting concept-artefact-representation in a more eclectic multi-modal way.

Using a multimodal approach has the potential of supporting children from less affluent backgrounds who may well have a more limited linguistic experience by the time they transfer to secondary school. However, using diagrams and other visual representation is not instinctive, but requires and responds to explicit instruction. Hence teachers can support learners in STEM by focussing more on representations as carriers of concepts by fostering in learners the skills in constructing and using visuospatial representations.

We still have a lot to learn about how our brain works with text and graphics, and whether there is a "best" way of using them in fostering learning. What we do know however is that it is advantageous taking a different look at our pedagogy, and exploring ways of offering a visual channel of communication. This would mean incorporating diagrams, and other graphics into our pedagogy not merely as visual *representations* to illustrate, but also as visual *forms* for the learner to activate and interpret and where the learner creates the visual as a way of strengthening their conceptual architecture. However, it means going further than this. It also means using *spatial representations* more widely as a mode of communication and cognition.

This might include concept mapping, worked examples, writing frames, etc. - though it will not be restricted to these.

As a conclusion I want to suggest we recognise and respect the visual and diagrammatical form as a pedagogical tool to represent STEM concepts and to work on STEM processes. We still do not have an understanding of the way in which diagrammatic competence develops over time, maybe because we have little idea of what we mean by diagrammatic competence in learning and have rarely used it as a legitimate pedagogical device within the classroom. We all have a lot to learn – even me, who has written a chapter praising and encouraging visualisation - with only four diagrams. In the words of one of my teachers in the 60s: "*Peter could do much better if he applied himself more*."

The geometrical property is – the four angle bisectors of any quadrilateral form a cyclic quadrilateral. IKEA are suggesting you should not tighten up the screw, or you won't get the back in!

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